October 8

Get Clickers and whiteboards

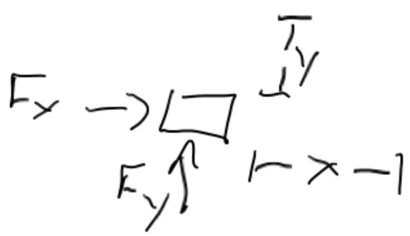
Energy and Work

Change in momentum = force times time

$\Delta \vec{p} = \vec{F}_{net} \Delta t$

Change energy = force times displacement (work)

$\Delta E = W$



$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$

Practice with scalar product

$\vec{r}_i = \langle 3, 7, 4 \rangle\, m \qquad \vec{r}_f = \langle 2, 10, 12 \rangle\, m$

$\vec{F} = \langle 10, -20, 30 \rangle\, N \qquad w = ?$

$\Delta \vec{r} = \vec{r}_f - \vec{r}_i = \langle 2-3,\ 10-7,\ 12-4 \rangle\, m$

$= \langle -1, 3, 8 \rangle\, m$

$W = F_x \Delta x + F_y \Delta y + F_z \Delta z = (10(-1)) + (-20(3)) + (30(8))$

$= 170\ Nm = 170\ J$

$w = \vec{F} \cdot \Delta \vec{r}$

$= |\vec{F}|\,|\Delta \vec{r}| \cos \theta$

Tangible: Ain't it a Drag?

$$\Delta \vec{p} = \vec{F}_{net} \Delta t$$

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} = .35\ N$$

$\theta = 45°$

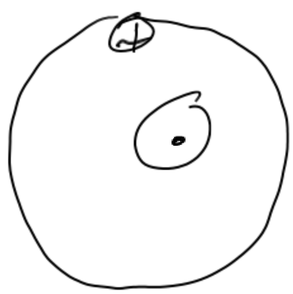
$\rightarrow$ 20cm

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

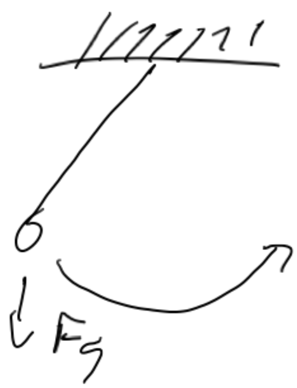
$$W = |\vec{F}||\vec{r}| \cos\theta = .1N(.2m)\cos 45°$$

$$= .014 J = 14 mJ$$

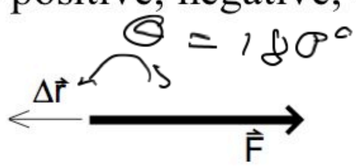
$\vec{w} = \vec{F} \cdot \Delta\vec{r} \qquad \Delta\vec{r} = 0 \Rightarrow w = 0$

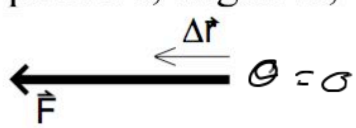


$$F = -G \frac{M_{\odot} m_{\oplus}}{r^2}$$

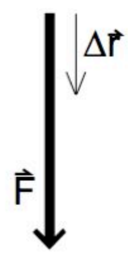


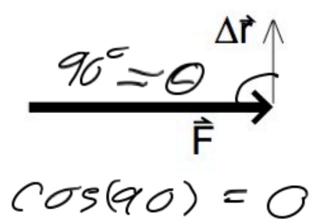
Clicker Questions

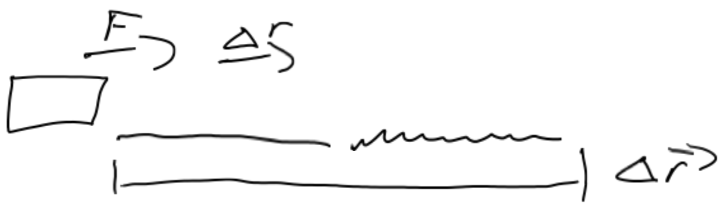
1. The diagram shows a force acting on an object and the displacement of the object while the force acts. Is the work done positive, negative, or zero? 	A) positive B) negative C) zero

2. The diagram shows a force acting on an object and the displacement of the object while the force acts. Is the work done positive, negative, or zero? 	A) positive B) negative C) zero

3. The diagram shows the force acting on an object and the displacement of the object while the force acts. Is the work done positive, negative, or zero? 	A) positive B) negative C) zero

4. The diagram shows a force acting on an object and the displacement of the object while the force acts. Is the work done positive, negative, or zero?  	A) positive B) negative C) zero

5. The diagram shows a force acting on an object and the displacement of the object while the force acts. Is the work done positive, negative, or zero?  	A) positive B) negative C) zero



$$W = \vec{F}_s \cdot \Delta\vec{r}_s + \vec{F}_r \cdot \Delta\vec{r}_r$$

$$W = \vec{F} \cdot \Delta\vec{r}$$

$$\Delta\vec{p} = \vec{F}_{net}\,\Delta t$$

Where does Energy come from?

$$\Delta E = \vec{F} \cdot \Delta \vec{r} \Rightarrow dE = F_x dx = \frac{dp_x}{dt} dx$$

$$\frac{dE}{dx} = \frac{dp_x}{dt} \qquad \vec{p} = \gamma m \vec{v} = \frac{m\vec{v}}{\sqrt{1 - |v|^2/c^2}}$$

$$E = \gamma m c^2 = \frac{1}{\sqrt{1 - |v|^2/c^2}} m c^2$$

$$[E = mc^2] \text{ rest energy}$$

Kinetic Energy

$$E_t = E_{rest} + E_{motion}$$

$$\gamma mc^2 = mc^2 + K \leftarrow \text{Kinetic Energy}$$

$$K = \gamma mc^2 - mc^2 = (\gamma - 1) mc^2$$

$$K = E - E_{rest} \quad \therefore \quad E = mc^2 = K$$

$$mc^2 = \gamma mc^2 - mc^2$$

$$1 = \gamma - 1 \Rightarrow 2 = \gamma = \frac{1}{\sqrt{1 - |v|^2/c^2}}$$

$$4 = \frac{1}{1 - |v|^2/c^2} \Rightarrow 4 - 4\frac{|v|^2}{c^2} = 1 \Rightarrow 3 = 4\frac{|v|^2}{c^2}$$

$$|v| = \sqrt{0.75}\, c = 0.86\, c = 2.6 \times 10^8\ m/s$$

Kinetic Energy at Low Speeds

$$(1+\varepsilon)^n = 1 + \frac{n}{1}\varepsilon + \frac{n(n-1)}{2\times 1}\varepsilon^2 + \frac{n(n-1)(n-2)}{3\times 2\times 1}\varepsilon^3 + \cdots$$

$$\left(1-\left[\frac{|\vec{v}|}{c}\right]^2\right)^{-\frac{1}{2}} = 1 + \frac{\left(-\frac{1}{2}\right)}{1}\left(-\left[\frac{|\vec{v}|}{c}\right]^2\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2\times 1}\left(-\left[\frac{|\vec{v}|}{c}\right]^2\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3\times 2\times 1}\left(-\left[\frac{|\vec{v}|}{c}\right]^2\right)^3 + \cdots$$

$$\approx 1 + \frac{1}{2}\left[\frac{|\vec{v}|}{c}\right]^2 + \frac{3}{8}\left[\frac{|\vec{v}|}{c}\right]^4 + \frac{5}{16}\left[\frac{|\vec{v}|}{c}\right]^6 + \ldots$$

$$\left[\approx 1 + \frac{1}{2}\left[\frac{|\vec{v}|}{c}\right]^2\right]$$

$$\gamma = \frac{1}{\sqrt{1-|v|^2/c^2}}$$

$\frac{v}{c} = 0.01$ $\gamma = 1.0005$ approx $\simeq 1.0005$

$\frac{v}{c} = 0.999$ $\gamma = 22.4$ approx $= 1.499$

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} = \left(1-\left[\frac{|\vec{v}|}{c}\right]^2\right)^{-\frac{1}{2}} mc^2 \approx \left(1+\frac{1}{2}\left[\frac{|\vec{v}|}{c}\right]^2\right)mc^2 = mc^2 + \frac{1}{2}m|\vec{v}|^2$$

$$K = \frac{1}{2}m|\vec{v}|^2 = \frac{|\vec{p}|^2}{2m} \qquad \vec{p} = m\vec{v}$$

Clicker questions

Q6: A ball whose mass is 2 kg travels at a velocity of < 0, –3, 4> m/s. What is the kinetic energy of the ball?	A) < 0, –6, 8 > J B) < 0, –3, 4 > J C) 2 J D) 10 J E) 25 J

$$K \simeq \frac{1}{2} m v^2 = \frac{p^2}{2m} \qquad v^2 = (-3)^2 + (4)^2$$

$$= \frac{1}{2}\, 2\,\text{kg}\, (9+16)\, \frac{\text{m}^2}{\text{s}^2} = 25\ \text{J}$$

Q7: A ball whose mass is 2 kg travels at a velocity of < 0, –3, 4> m/s. What is the rest energy of the ball?	A) 0 J B) 25 J C) 6e8 J D) 9e16 J E) 1.8e17 J

$E_{rest} = mc^2 = 2\,kg\,(3\times10^8\,m/s)^2 = 1.8e17\,J$

Q8: Consider an electron (mass 9e-31 kg) moving with speed v = 0.9c. What is its rest energy?	A) 7.3e-31 J B) 8.1e-14 J C) 1.05e-13 J D) 1.86e-13 J E) 2.7e8 m/s

$E_{rest} = mc^2$

Q9: Consider an electron (mass 9e-31 kg) moving with speed v = 0.9c. What is its total (particle) energy?	A) 7.3e-31 J B) 8.1e-14 J C) 1.05e-13 J D) 1.86e-13 J E) 2.7e8 m/s

$$E = \gamma mc^2 = \frac{1}{\sqrt{1-(.9)^2}}\, 9e^{-31}\,\text{kg}\,(3\times10^{8}\ \text{m/s})^2$$

$$= 1.86e^{-13}\ \text{J}$$

Q10: Consider an electron (mass 9e-31 kg) moving with speed v = 0.9c. Its rest energy is 0.8e-13 J, and its (total) particle energy is 1.86e-13 J. What is its kinetic energy?	A) 7.3e-31 J B) 3.28e-14 J C) 8.1e-14 J D) 1.06e-13 J E) 1.86e-13 J

$E_T = E_{rest} + K$

Work done by non-constant forces

$$W = \vec{F} \cdot \Delta\vec{r}$$

$$\frac{\Delta p}{\Delta t} = \frac{dp}{dt}$$

$$= \int_i^f \vec{F} \cdot d\vec{r}$$

VPython simulation of a spacecraft flyby